Flawed Applications of Bernoulli's Principle

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Flawed Applications of Bernoulli's Principle

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One of the most popular demonstration experiments pertaining to Bernoulli's principle is the production of a water spray by using a vertical plastic straw immersed in a glass of water and a horizontal straw to blow air towards the top edge of the vertical one. A more general version of this phenomenon, appearing also in school physics problems, is the determination of the rise of the water level h in the straw (see Fig. 1).

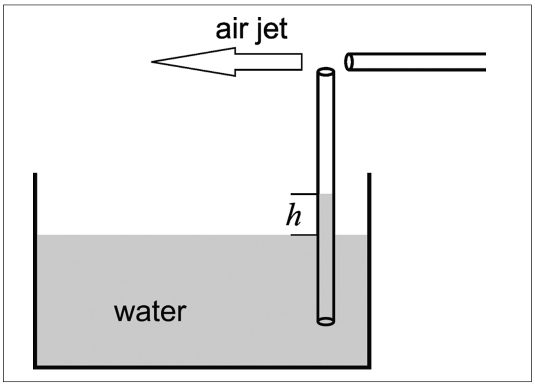

Fig. 1. Blowing air over the vertical straw causes a rise of the water level by *h* cm.

The problem

The problem is usually stated as follows: "What is the required speed of the air stream v so that the water level in the straw rises by h cm?" In our country, similar experimental activities and problems are included in the high school curriculum[1] and are also required subject matter for the national entry examinations for universities and polytechnic schools. These problems are usually solved by applying Bernoulli's equation at two points, one in the air stream just above the vertical straw and the other in a position within the straw where the air is not moving and pressure is normal. This argument leads to the relation $(1/2)\, d_{\text{air}}\, v^2 = d_{\text{water}} gh$, which when solved for v provides the answer, given the densities of air and water.

Similar problems, accompanied by similar explanations of the phenomenon, can be found not only in physics textbooks, but also in scientific journal articles.[2,3] Following the arguments in the literature, the water level rise in the straw depends only on the speed of air stream above it. In the following, we attempt to investigate the validity of these arguments and design an experimental demonstration using everyday materials, reproducible in most school environments. This choice led us to avoid experiments in an air tunnel, even one made with commonly found objects (see, for example, the one presented in https://www.apogeerockets.com/education/downloads/Newsletter252.pdf).

As will be explained in the following, the use of more elaborate instrumentation like air tunnels and flat or curved airfoils usually used in aerodynamics experiments would not change the observed results.

Experimental investigation of the validity of explanations based on Bernoulli's equation

• ***Experiment 1:*** Air flowing on top of a thin flat foil in parallel with it.

As shown in Fig. 2(a), the device used consists of a thin transparent plastic tube attached to a hole in the middle of a thin flat piece of balsa wood. The tube is held in place with the help of a small drilled wooden cube, which is glued on the bottom face of the balsa wood. The other end of the tube is connected to a manometer, which could be an electronic one or an open U-tube water manometer. The manometer consists of the long end of the same plastic tube. The open end of the manometer preferably has a small inclination, so that the water displacement becomes more evident.[4]

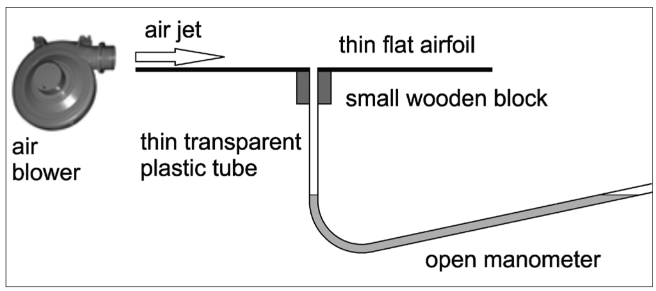

Fig. 2. (a) Constant air flow over the top end of the tube.

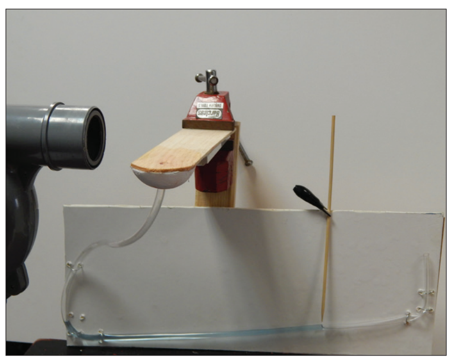

Fig. 2. (b) During the operation of the air blower, there is no change of the water level in the manometer.

For the production of the air flow, we used a cheap air blower used for cleaning electric appliances (Muller Q1F-2.8, electrical power 500 W, costing under $20). The air blower is fixed so the air flows above and in parallel to the balsa wood. Special care has been taken so that the top end of the plastic tube is flush to the top surface of the balsa wood [see Fig. 2(b)]. In case we want to investigate the production of a water spray, we simply insert the bottom end of the tube in a bottle filled with water and bring the rim of the bottle neck as close to the balsa wood as possible.

When we turn the air blower on, despite the predictions based on Bernoulli's law, the experiment shows that there is no change in the water level in the manometer (indicated by the end of a thin wooden stick), and therefore no change of pressure in the plastic tube.

The lack of production of under-pressure in a similar experimental setting has also been shown by Kamela.[5]

- ***Experiment 2:*** Air flowing over a curved airfoil.

Leaving all other devices in place, we replace the flat airfoil with a curved one by flipping the balsa wood board and reinserting the plastic tube flush to the surface, as shown in Fig. 3. When we turn on the air blower, a water level rise in the tube is observed, as can be attested by the decrease of the water level in the manometer.

It should be noted here that in order to produce the curved airfoil surface, we glued half a Styrofoam sphere (diameter ~ 4 cm) on the balsa wood board and drilled it carefully to match the diameter of the plastic tube.

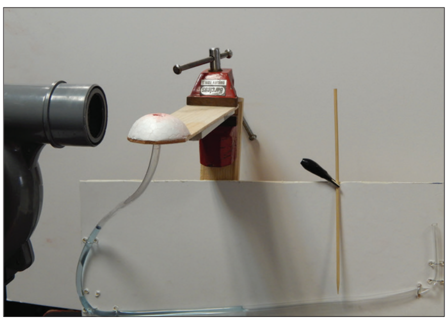

Fig. 3. When the curved airfoil is used, there is a rise in the water level.

As expected, if we immerse the plastic tube into a bottle full of water placed under the balsa wood and we turn the air blower on, we will observe the production of a water spray.

Discussion of the experimental results

In the first experiment we disproved the production of air under-pressure at a point above the top end of the tube. This means that the widely spread view that the higher air flow speed above the tube is sufficient to cause an under-pressure relative to a point near the surface of water due to Bernoulli's principle is false. Similar experiments have been proposed in the literature showing that the pressure in an unobstructed stream of air (e.g., over an obstruction parallel to the streamlines) is equal to the atmospheric one.[4-6]

In the second experiment we confirmed the formation of under-pressure in the top of the tube. How should this be explained?

Interpretation of the results of Experiments 1 and 2

In the following, we will attempt to explain the results of Experiments 1 and 2 according to a three-step causal sequence, based on the phenomenological analysis of the flow before, over, and after an obstacle, as well as a more formal treatment.

In the first case [Experiment 1, see Fig. 4(a)], where there is a flow above the horizontal flat surface, the streamlines are straight lines parallel to the surface (in reality, in order for this assumption to be valid, the use of an air tunnel is necessary). In the second case [Experiment 2, see Fig. 4(b)], we distinguish the following regions of interest regarding air flow:

a) Flowing from the left, up to point 1, the air stream moves undisturbed; therefore, the air pressure in this area is the same as the pressure of its surroundings, i.e., the atmospheric one ($P_{\text{free stream}} = P_{\text{ambient}}$).[4-6]

b) After point 1, the air stream encounters the obstacle, where:

b1) In the region between points 1 and 2, the air is forced to flow upwards and then over the curved surface, following the shape of the curve, according to the so-called Coanda effect (an air stream flowing along a smooth curved surface tends to follow its shape, due to air viscosity).[4,7] Therefore, the streamlines over the curved surface are also following its shape. In this region the air velocity is reduced while its pressure is increased, becoming higher than the atmospheric one. Since the pressure outside the air stream is equal to the atmospheric one, there is an upwards gradual reduction of the pressure (pressure gradient), which leads to concave up streamlines (**Step 1:** Deformation of the streamlines).

b2) In the region 2-3, according to the Coanda effect, the forced air flow above the obstacle causes the reduction of air pressure over the open end of the tube, establishing a pressure gradient ΔP both radially (lower towards the center of the curved streamlines) and along the stream flow (**Step 2:** Creation of under-pressure gradients). This pressure gradient causes the air stream to accelerate towards the region of low pressure, leading to convex up streamlines, which are perpendicular to both the radial ΔP and the higher velocity air flow [**Step 3:** Acceleration of the stream flow, see Fig. 4(b)].

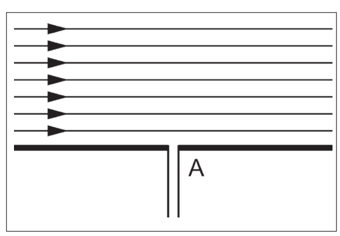

Fig. 4. (a) Straight streamlines.

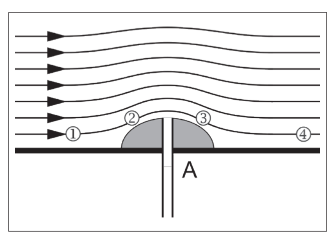

Fig. 4. (b) Curved streamlines.

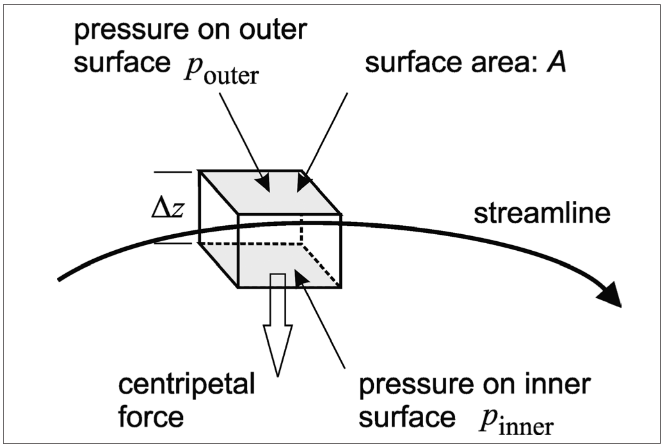

Fig. 5. The centripetal force on an elementary air volume, perpendicular to the curved streamline.

c) The air flow in region 3-4 can be described in the same way as for region 1-2, leading to concave up streamlines.

d) Finally, the region after point 4 can be considered as undisturbed, leading to straight streamlines, as in the region before point 1.

Considering an elemental volume of air within the stream in the region 2-3 of Fig. 4(b), the pressure at a point of the outer side of the volume (p_{outer}) is higher than the pressure at a point of the inner side (p_{inner}), see Fig. 5. Therefore,

$$p_{outer} > p_{inner}$$

and

$$p_{outer} = p_{inner} + \Delta p, \text{where } \Delta p = \Delta z\left(\frac{dp}{dz}\right), \tag{1}$$

(dp/dz is the pressure gradient along the z-axis, perpendicular to the streamline at this position. The pressure is increasing outwards and at a certain distance from the obstacle, where the flow is undisturbed, becomes equal to the atmospheric one).

Due to this pressure difference between the outer and the inner side of the elemental volume of air within the stream, since it is moving along a curved and not a straight line, we must consider the existence of an additional force perpendicular to the direction of motion, i.e., a centripetal force (see Fig. 5).

The centripetal force F acting on the elementary air volume:

1. is due to the pressure difference Δp at the inner and outer sides of the elementary volume and is equal to $F = -A\,\Delta p$. Since, according to Eq. (1),

$$\Delta p = \Delta z\left(\frac{dp}{dz}\right), \text{we obtain}$$
$$F = -A\Delta z\left(\frac{dp}{dz}\right). \tag{2}$$

The negative sign in Eq. (2) indicates that the force is directed along the negative z-axis.

2. is given by the relation

$$F = -\Delta m\frac{v^2}{R}, \tag{3}$$

where Δm is the mass of the elementary volume of air, given by the relation $\Delta m = \rho A \Delta z$, where ρ is air density and R is the local curvature radius. The centripetal force is directed towards the curvature center of the streamline at that point; therefore, it is directed downwards [hence the minus sign in Eq. (3)]. From Eqs. (2) and (3) we conclude that

$$\Delta m\frac{v^2}{R} = A\Delta z\left(\frac{dp}{dz}\right) \rightarrow$$
$$\rho A\Delta z\frac{v^2}{R} = A\Delta z\left(\frac{dp}{dz}\right) \rightarrow \frac{dp}{dz} = \rho\frac{v^2}{R}. \tag{4}$$

According to Eq. (4), curved streamlines are related to an air pressure gradient, where pressure is increased from inside towards outside, along the radius of curvature. This pressure gradient is justified by the three causal steps described above, and is responsible for both the higher air velocity and the curved streamlines.

Now, according to the relation

$$\frac{dp}{dz} = \rho\frac{v^2}{R},$$

we conclude that if $R \rightarrow \infty$, i.e., if the streamline is straight, then $v^2/R \rightarrow 0$; therefore, also $dp/dz \rightarrow 0$, resulting in zero pressure gradient perpendicular to the flat streamlines. This is precisely why we observed no rise of water level in Experiment 1.

Our analysis agrees with Weltner,[4] who argues,

> "As a rule, physics textbooks neglect the treatment of normal acceleration of fluids in curved streamlines. Thus they omit to discuss the pressure gradients normal to the velocity if streamlines are curved …. The neglect of pressure gradients related to curved streamlines is disastrous because the mechanism producing low pressure in streaming fluids is thus made impossible to be understood …. Obstacles cause curved streamlines and generate pressure gradients of air and thus cause regions of higher or lower pressure."

Applying Newton's second law for the tangential force acting on the elementary volume of air in Fig. 5 leads to the well-known Bernoulli's law, with the added benefit of showing clearly that it's not the higher air velocity that causes the reduced pressure but the lower pressure that causes the higher velocity.[4,6]

A similar analysis and application of Newton's second law to the phenomenon of airfoil lift also provides the correct causal link and justifies the reasoning that Δp causes Δv.[8-10]

Explanation of the observed rise of the water level in the straw

The air flow when blowing above the upper end of a vertical straw immersed in water, and specifically the shape of the streamlines before, over, and after the obstruction, can be described using the same principles as for Fig. 4(b) (although the surface of the obstacle is not a smooth curved one and consequently the streamlines are not shaped exactly this way.)

In the so-called "water spray" demo, the under-pressure created above the upper end of the vertical straw (see region 2-3 of Fig. 6) is not due to an increased air velocity, as implied in journal articles[3] or physics textbooks.[1,2] The under-pressure is caused by the three-stage causal sequence described previously in the paragraphs explaining the shape of streamlines in Fig. 4(b). It should be noted here that the production of a water jet or the rise of the water level in the manometer can also be observed in the context of Experiment 1 [see Fig. 2(a)], provided the plastic tube protrudes over the flat balsa wood surface, thus distorting the streamlines around it.

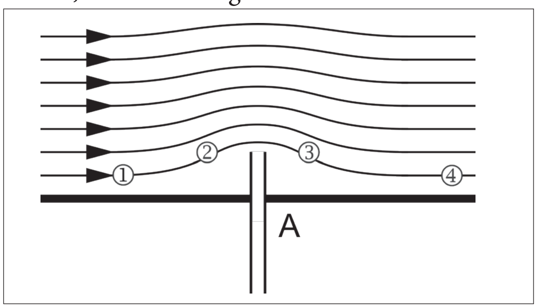

Fig. 6. Curvature of the streamlines before, during, and after the obstacle.

Conclusions

The widespread view that the pressure within a stream of moving air is always lower than in the stationary surrounding air is encountered not only in school textbooks but also university/college textbooks[11] and even in scholarly articles.[3,12] In 1972, Smith[13] observed that

> "[m]illions of children in science classes are being asked to blow over curved pieces of paper and observe the fact that the paper 'lifts,' or they are asked to blow between two suspended apples and to observe that the apples are drawn together … . They are also told that Bernoulli's theorem is responsible for lift on the airplane wing."

The common explanation of the above-mentioned phenomena is usually based on the claim that the higher air velocity in a specific region of flow causes the pressure to drop, according to Bernoulli's equation. In this article we attempt to demonstrate that this explanation is flawed and that the observed under-pressure is caused by a three-step causal sequence:

1. The stream flow, when obstructed by an obstacle, is forced to curve and move along its surface, due to air viscosity (Coanda effect[4,7,14]).
2. The deviation of the air stream along the obstacle causes an under-pressure gradient both radially and along the direction of its movement.
3. The stream then accelerates towards the region of low pressure, leading to the observed curvature of streamlines and the higher velocity flow.

In conclusion, the observed higher velocity flow is the result of the lower pressure and not its cause.

Acknowledgment

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